Rocket Equations

m_R = rocket mass in kg
m_E = engine mass (including propellant) in kg
m_P = propellant mass in kg
a = acceleration m/s^2
F = force in kg · m/s^2
g = acceleration of gravity = 9.81 m/s^2
A = rocket cross-sectional area in m^2
c_d = drag coefficient = 0.75 for average rocket
ρ = air density = 1.223 kg/m^3
τ = motor burn time in seconds
T = motor thrust in Newton i.e. in kg · m/s^2
I = motor impulse in Newton · seconds
v_τ = burnout velocity in m/s
h_B= altitude at burnout in m
h_C= coasting distance in m

$m_B = m_R + m_E - m_P/2$	boost mass in kg
$m_C = m_R + m_E - m_P$	coast mass in kg
$m_B \cdot g$	boost gravity force in kg · m/s^2
$m_C \cdot g$	coast gravity force in kg · m/s^2
$k = \frac{1}{2} \cdot \rho \cdot c_d \cdot A$	air drag coefficient in kg/m
$k \cdot v^2$	air resistance in kg · m/s^2
$\tau = \frac{I}{T}$	burnout time in seconds

Boost Phase: Burnout Time,Velocity and Altitude

$$
\begin{aligned}
F &= m \cdot a \\
&= m \cdot \frac{dv}{dt} \\
&= T - mg - k \cdot v_\tau^2
\end{aligned}
$$

$$m \cdot \frac{dv}{dt} = T - mg - k \cdot v_\tau^2$$

$$
\begin{aligned}
dt &= \frac{m \cdot dv}{T - mg - k \cdot v_\tau^2} \\
&= \frac{m \cdot dv}{k \cdot \frac{T - mg}{k} - k \cdot v_\tau^2} \\
&= \frac{m \cdot dv}{k \cdot q^2 - k \cdot v_\tau^2} \\
&= \frac{m}{k} \cdot \frac{dv}{q^2 - v_\tau^2}
\end{aligned}
$$

$$q^2 = \frac{T - m_B \cdot g}{k}$$

$$q = \sqrt{\frac{T - m_B \cdot g}{k}}$$

$$\tau = \frac{m}{k} \cdot \int_0^{v_\tau} \frac{dv}{q^2 - v_\tau^2}$$

$$\boxed{\tau = \frac{m_B}{k} \cdot \frac{1}{2q} \cdot \ln\left(\frac{q + v_\tau}{q - v_\tau}\right)}$$

$$\frac{2kq}{m_B} \cdot \tau = \ln\left(\frac{q + v_\tau}{q - v_\tau}\right)$$

$$\frac{2kq}{m_B} = p$$

$$p \cdot \tau = \ln\left(\frac{q + v_\tau}{q - v_\tau}\right)$$

$$-p \cdot \tau = \ln\left(\frac{q - v_\tau}{q + v_\tau}\right)$$

$$e^{-p\cdot\tau} = \frac{q - v_\tau}{q + v_\tau}$$

$$(q + v_\tau) \cdot e^{-p\cdot\tau} = q - v_\tau$$

$$q \cdot e^{-p\cdot\tau} + v_\tau \cdot e^{-p\cdot\tau} = q - v_\tau$$

$$v_\tau + v_\tau \cdot e^{-p\cdot\tau} = q - q \cdot e^{-p\cdot\tau}$$

$$v_\tau \cdot \left(1 + e^{-p\cdot\tau}\right) = q \cdot \left(1 - e^{-p\cdot\tau}\right)$$

$$\boxed{v_\tau = q \cdot \frac{1 - e^{-p\cdot\tau}}{1 + e^{-p\cdot\tau}}}$$

burnout velocity

$$\begin{aligned} F &= m \cdot a \\ &= m \cdot \frac{dv}{dt} \\ &= m \cdot \frac{dv}{dh} \cdot \frac{dh}{dt} \\ &= m \cdot v \cdot \frac{dv}{dh} \\ &= T - mg - k \cdot v_\tau^2 \end{aligned}$$

$$\frac{dh}{dt} = v$$

$$q^2 = \frac{T - m_B \cdot g}{k}$$

$$q = \sqrt{\frac{T - m_B \cdot g}{k}}$$

$$m \cdot v \cdot \frac{dv}{dh} = T - mg - k \cdot v^2$$

$$dh = \frac{m \cdot v}{k \cdot q^2 - k \cdot v^2} \cdot dv$$

$$dh = \frac{m}{k} \cdot \frac{v}{q^2 - v^2} \cdot dv$$

$$h = \frac{m}{k} \cdot \int_0^{v\tau} \frac{v \cdot dv}{q^2 - v^2}$$

$$h_B = \frac{m_B}{k} \cdot \frac{1}{2} \cdot \left[\ln\left(q^2 - v^2\right)\right]_0^{v\tau}$$

$$h_B = \frac{m_B}{2k} \cdot \left[\ln\left(q^2\right) - \ln\left(q^2 - v_\tau^2\right)\right]$$

$$\boxed{h_B = \frac{m_B}{2k} \cdot \ln\left(\frac{q^2}{q^2 - v_\tau^2}\right)}$$

burnout altitude

$$h_B = \frac{m_B}{2k} \cdot \ln\left(\frac{T - m_B \cdot g}{T - m_B \cdot g - k \cdot v_\tau^2}\right)$$

Coast Phase: Altitude and Time

$$dh = \frac{m_C \cdot v \cdot dv}{-m_C \cdot g - k \cdot v^2}$$

$$dh = m_C \cdot \frac{v \cdot dv}{k \cdot \frac{-m_C \cdot g}{k} - k \cdot v^2}$$

$$\frac{-m_C \cdot g}{k} = q_C^2$$

$$dh = \frac{m_C}{k} \cdot \frac{v \cdot dv}{q_C^2 - v^2}$$

$$h_C = \frac{m_C}{k} \cdot \int_{v\tau}^{0} \frac{v \cdot dv}{q_C^2 - v^2}$$

$$\boxed{h_C = \frac{m}{2k} \cdot \ln\left(\frac{q_C^2 - v_\tau^2}{q_C^2}\right)}$$

coast altitude

$$h_C = \frac{m}{2k} \cdot \ln\left(\frac{m_C \cdot g + k \cdot v_\tau^2}{m_C \cdot g}\right)$$

$$F = m \cdot (-a)$$

$$= m \cdot \left(-\frac{dv}{dt}\right)$$

$$= T - m_C \cdot g - k \cdot v_\tau^2$$

$$dt = \frac{-m_C \cdot dv}{-m_C \cdot g - k \cdot v^2}$$

$$dt = m_C \cdot \frac{dv}{m_C \cdot g + k \cdot v^2}$$

$$q_a^2 = \frac{m_C \cdot g}{k}$$

$$t_C = \frac{m_C}{k} \cdot \int_{v\tau}^{0} \frac{dv}{q_a^2 + v^2}$$

$$\boxed{t_C = \frac{m_C}{k} \cdot \frac{1}{q_a} \cdot \arctan\left(\frac{v_\tau}{q_a}\right)}$$

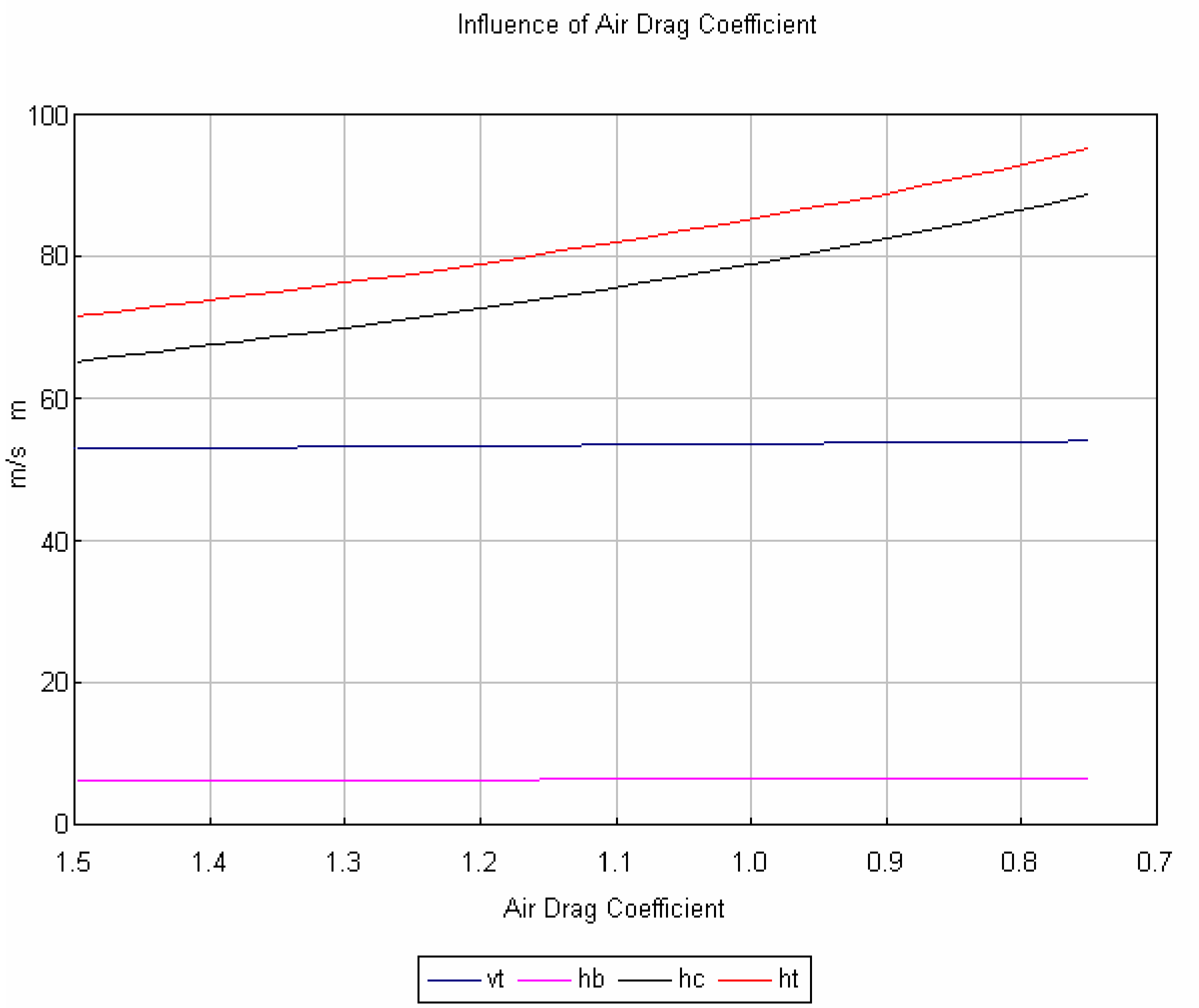
time from v_τ to 0

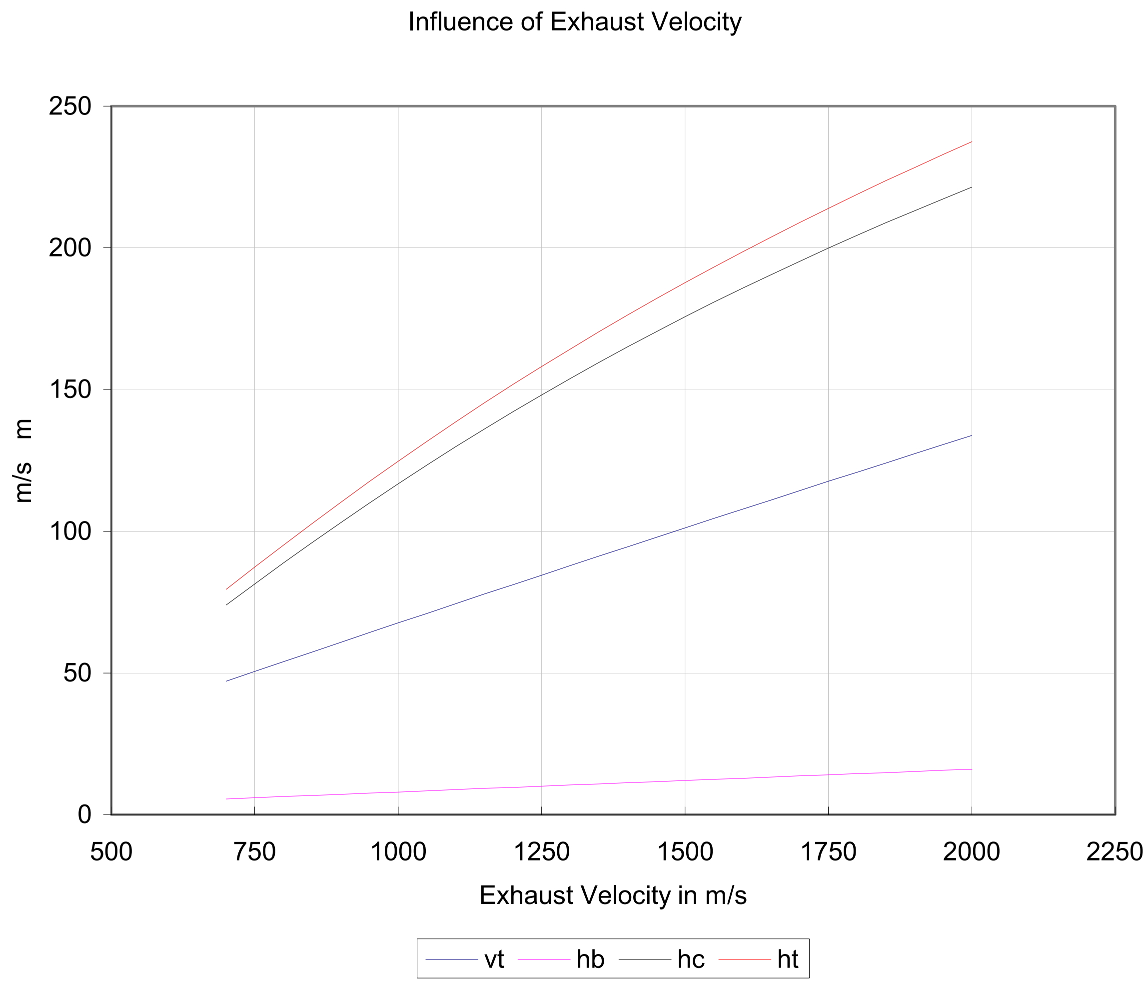
$$t_C = \sqrt{\frac{m_C}{k \cdot g}} \cdot \arctan\left(\frac{v_\tau}{\sqrt{\frac{m_C \cdot g}{k}}}\right)$$

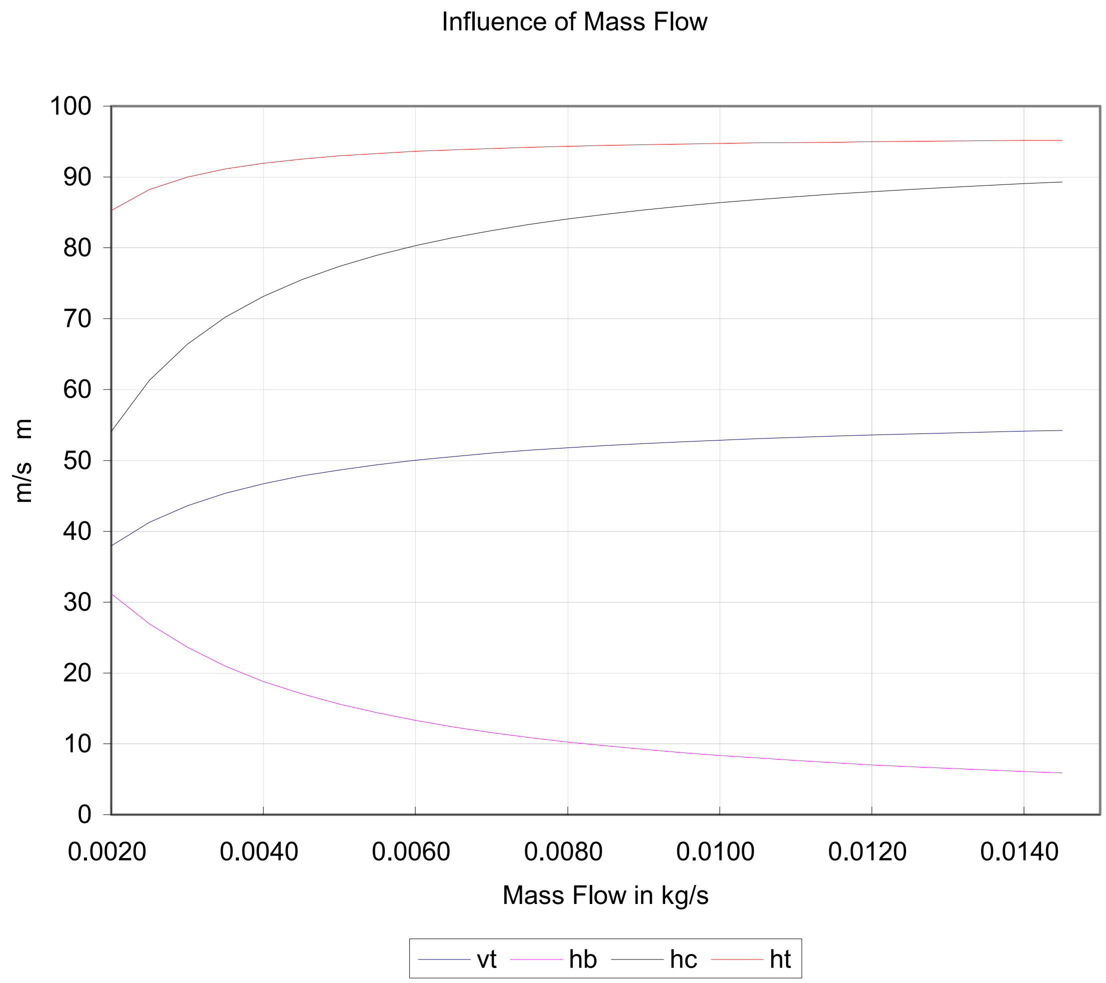
$$q_a = \sqrt{\frac{m_C \cdot g}{k}}$$

ROCKET EQUATIONS

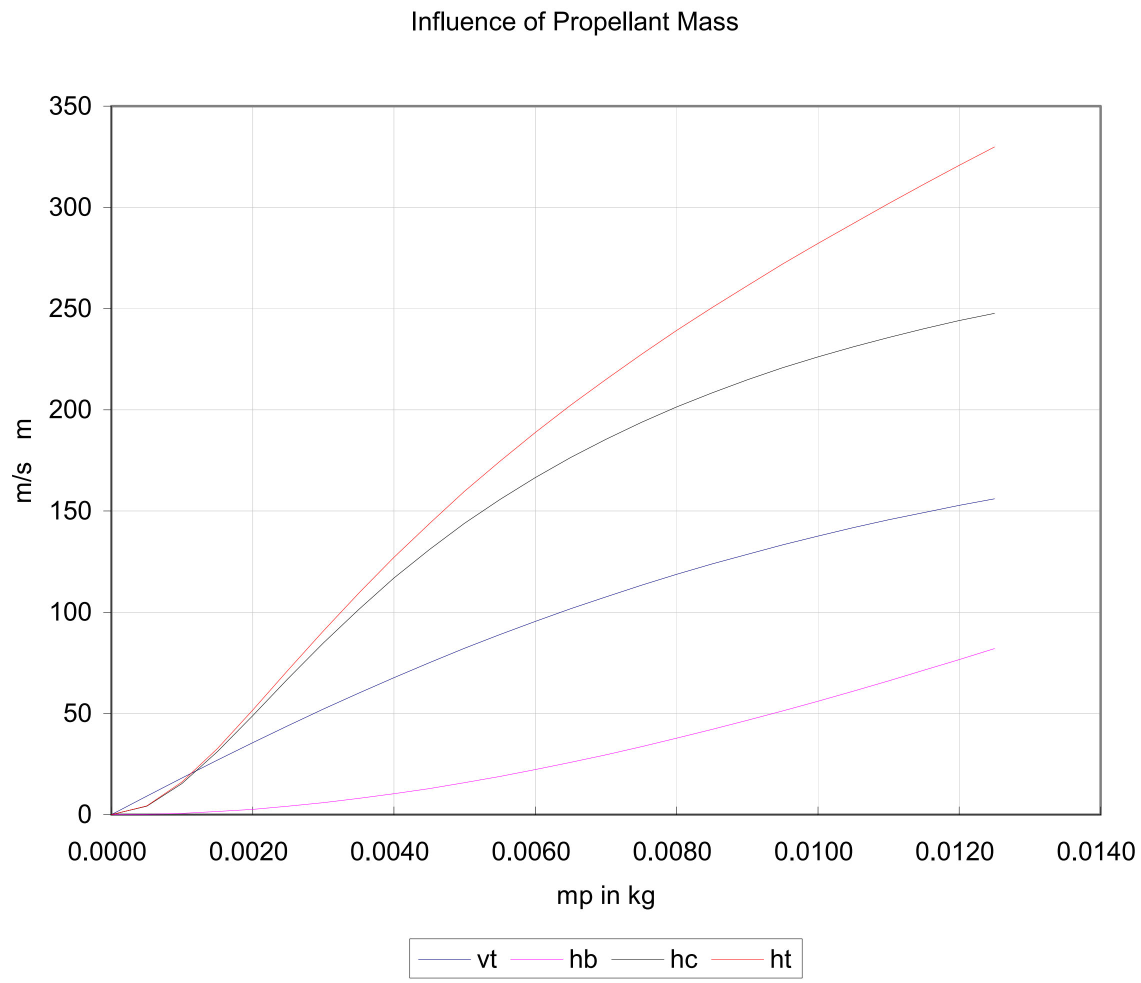
gravit accel	g	9.8100	m/s²	thrust	T	10.6829	N=kg*m/s²
air density	rho	1.2230	kg/m^3	impulse	I	2.4960	N*s=kg*m/s
drag coef	cd	0.7500		boost mass	mb	0.04344	kg
rocket body	mr	0.0288	kg	coast mass	mc	0.04188	kg
engine empty	ee	0.0131	kg	burnout time	tau	0.23364	s
propellant	mp	0.0031	kg	velocity b	vt	53.97011	m/s
rocket total	mt	0.0450	kg	altitude b	hb	6.37419	m
engine init	me	0.0162	kg	altitude c	hc	88.73626	m
propellant	p%	6.9333	%	altitude t	ht	95.11045	m
mass flow	mü	0.0134	kg/s	coast time	tc	3.92459	s
exhaust v	vex	800.0000	m/s	max velocity	vkm/h	194.29240	km/h
diameter	d	0.0250	m	max air drag	Fd	0.65574	N=kg*m/s²
c-s-area	A	0.00049	m²	check			
drag factor	k	0.00023	kg/m	burnout time	tau	0.23364486	s
	q	213.4473	m/s				
	qc²	-1824.93776					
	qa	42.7193	m/s				
	p	2.2124	1/s				




Influence of Exhaust Velocity
250
200
150
100
50
0
m/s m
500
750
1000
1250
1500
1750
2000
2250
Exhaust Velocity in m/s
vt
hb
hc
ht



Influence of Mass Flow
100
90
80
70
60
50
40
30
20
10
0
m/s m
0.0020
0.0040
0.0060
0.0080
0.0100
0.0120
0.0140
Mass Flow in kg/s
vt
hb
hc
ht


Influence of Propellant Mass
350
300
250
200
150
100
50
0
m/s m
0.0000
0.0020
0.0040
0.0060
0.0080
0.0100
0.0120
0.0140
mp in kg
vt
hb
hc
ht